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A Decoherence-Reduction Scheme by Waveguides in Quantum Information Processing

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Abstract Based on the result of cavity quantum electrodynamics, we suggest a method, in which the Fabry-Perot cavity or the confocal cavity is replaced by a waveguide with the size comparable to the wavelength of the photon, to reduce decoherence caused by spontaneous emission in quantum information processing, especially in the realization of quantum computation. Since a waveguide has a lowest cutoff frequency while a Fabry-Perot cavity or a confocal cavity has none, the spontaneous emission of excited atoms will be forbidden in an ideal waveguide with an appropriate size. To avoid the influence of the non-ideal conducting walls on the atom in a realistic waveguide, which will lead to decoherence, we suggest that the waveguide should be coated by a thin film of transparent insulating medium. In our method, the quantum information is represented by a multi-level atom or molecule; any two of its levels can be used to represent a qubit in principle. Our method greatly extends the choice of the material to be used in the realization of quantum computation, and it can be used in most schemes to reduce the decoherence caused by spontaneous emission.

Keywords Quantum computation · Quantum information · Cavity QED · Waveguide · Spontaneous emission

1 Introduction

Quantum computation has attracted much interest since quantum computers are in principle able to solve hard computational problems more efficiently than present classical computer and can actually be realized. One kind of the schemes for realizing quantum computation is based on cavity quantum electrodynamics (cavity QED) [1–4], in which a single photon interacts with a single atom in a cavity. Nevertheless, the main obstacle in realizing a quantum computer is decoherence, which results from the coupling of the system with the environment. Among the sources of decoherence, the spontaneous emission of atoms is a

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very important one, especially in the realization of quantum computation with cavity QED. In the present paper, we will concentrate on the problem of how to reduce the decoherence caused by spontaneous emission, and propose a method to avoid such decoherence in quantum computation by using appropriate waveguides. Although there are some schemes, in which the problem of spontaneous emission is solved [5, 6], our method can be applied to most schemes in which the excited atoms may spontaneously emit and cause decoherence.

In the schemes of the realization of quantum computation with cavity QED, the cavity is used to confine photons and to transfer information between atom and cavity; it is often represented by a Fabry-Perot cavity [7] since an excited atom can resonate with a Fabry-Perot cavity and in principle photons can be confined in it. However, in experiments, the Fabry-Perot cavity is often replaced by a confocal cavity to confine the photons more efficiently [3], and the size of the cavity is usually much larger than the wavelength of the photon, e.g., in the scheme in [3], the size of the cavity is a few tens of micrometers. In principle, a Fabry-Perot cavity plays the same role as the confocal cavity in the realization of quantum computation since both of them can confine photons and resonate with the atom, so in this paper we will ignore the difference between these two kinds of cavities and regard both of them as Fabry-Perot cavities.

The behavior of quantum systems in confined space has been studied for many years in the field of cavity QED [8] and in statistical mechanics [9–16]. The spontaneous emission rate of atoms in a cavity can increase or decrease depending on the shape and size of the cavity [17, 18]. In confined space, the values of the photon momentum are discrete at certain directions and a non-zero lower limit exists due to the boundary, so the phase space volume of the final state is usually smaller than that in free space, and this leads to the decrease of the spontaneous emission rate of excited atoms. However, when the cavity resonates with the atom, the spontaneous emission will be extremely enhanced. The influence on the spontaneous emission rate is tightly related to the cavity geometry. In a Fabry-Perot cavity, which is often used in the schemes of quantum computation, the spontaneous emission rate of an atom depends on the polarization of the atom; except the case that the atomic dipole is exactly parallel to the surfaces of the cavity, the atom will always spontaneously emit. In a waveguide, however, the spontaneous emission can be eliminated as long as the frequency of the emitted photon is lower than the lowest cutoff frequency of the waveguide [18, 19]. This property can be used in the realization of quantum computation to avoid spontaneous emission.

In this paper, based on the calculation of spontaneous emission rate of excited atoms in ideal and non-ideal waveguides, we suggest to use a waveguide with a size comparable to the wavelength of the photon emitted by the atom in quantum information processing to avoid the spontaneous emission which causes decoherence. In quantum information processing, to avoid decoherence we require that (1) the lifetime of the excited atom should be as long as possible in information transmission, (2) the coupling between atom and cavity should be as strong as possible in information transfer. The strong coupling in the information transfer can be achieved by the resonance between atom and cavity, and what we will do is to extend the lifetime of excited atoms in information transmission. Our result will show that by choosing an appropriate waveguide, since the size of the waveguide will strongly influence the lifetime of the excited atom, both the two requirements can be achieved. Furthermore, our method has some other advantages. (1) In many models of quantum computation, the atom is regarded as a two-level one, but a realistic atom must be multi-level, the spontaneous emissions between the other levels will lead extra decoherence but are usually ignored. In our method, multi-level atoms can be used, which does not cause extra decoherence, since the spontaneous emission between the other levels is forbidden. (2) Usually, the choice of the material for representing the qubit is limited by the structure of the energy levels and the lifetimes of the excited states of the atom; the appropriate material is limited to the atom which can be approximately regarded as a two-level one. In our method, the choice of the material is greatly extended. In principle, we can choose any two levels of a multi-level atom to represent the qubit even if the origin lifetime of the excited state is short, because in an appropriate waveguide all the spontaneous emissions can be eliminated. (3) Since the spontaneous emission is forbidden in ideal waveguides, the distance of information transmission is extended, so there will be less restrictions in the design of the device of quantum computation.

In a realistic waveguide, however, the interaction between the atom and the non-ideal conducting walls of the waveguide will greatly change the result of the ideal case, especially when the atom is very close to the walls. When the distance between an atom and a non-ideal conducting surface, z, is small, the spontaneous emission rate will approximately increase as z^{-3}. In other words, a non-ideal conducting waveguide not only spoils the forbiddance of spontaneous emission in an ideal waveguide, but also leads to a very rapid spontaneous emission when the atom is close enough to one of the walls. This spontaneous emission must be eliminated to protect the coherence of quantum information. Fortunately, the massive increase in spontaneous emission occurs at only a narrow range close to the surface. For a metal waveguide this range is only about 5 percent of the wavelength at the visible light regime. Therefore, we suggest that the waveguides used to suppress the spontaneous emission should be coated by a thin film of transparent insulating medium at the inner surfaces. Then the atom cannot be too close to the surface so that the influence of the non-ideal conducting walls is remarkably reduced, and the spontaneous emission rate of the atom can be greatly suppressed.

The method we proposed here is not restricted to any concrete scheme of quantum computation; it is a general framework and can be applied to many different schemes. However, for describing our method more concrete, in this paper we will take a scheme of quantum computation with cavity QED as an example [2] to show how to use waveguides to avoid the decoherence caused by spontaneous emission. We hope that this method can help to overcome the obstacle in the realization of quantum computation.

This paper is organized as follows. In Sect. 2, we analyze the spontaneous emission of an excited atom in an ideal cylindrical waveguide based on the quantum field theory, and compare it with the spontaneous emission in free space. The result shows that the spontaneous emission is forbidden when the radius of the ideal waveguide is made sufficiently small. In Sect. 3, we consider the influence of a non-ideal conducting waveguide on the spontaneous emission of an excited atom. The result shows that the spontaneous emission will be greatly enhanced when the atom is very close to a surface, and the ranges of the influence for some metal materials are estimated. According to this result, in Sect. 4, we suggest that to use a waveguide in quantum information processing will further reduce the decoherence caused by the spontaneous emission, and for a realistic case, we can use the waveguide coated by a thin film of transparent insulating medium in its inner surfaces to avoid the spontaneous emission caused by the interaction between the atom and the walls. This method is a general framework to avoid decoherence in quantum computation, but is not restricted to any concrete scheme. Finally, some discussions are presented in Sect. 5.

2 Spontaneous Emission Rate of an Atom in an Ideal Waveguide

We first consider the problem of an excited atom in a waveguide with ideal conducting walls. The problem of spontaneous emission in a cylindrical waveguide has been considered in [18]. In that treatment some approximations are used: the transition amplitude is regarded as the same as that in free space; the mode of electromagnetic field in the cavity is treated as the plane wave. In this section, we will consider the effect of boundary on the spontaneous emission by the example of an atom in a cylindrical waveguide.

The spontaneous emission rate of an excited atom is determined by the transition amplitude between two states and the phase space volume of the final state. In a waveguide, both the two factors are different from those in free space: The transition amplitude is different because the modes of the electromagnetic field in a waveguide are restricted by the boundary; the phase space volume is different because some components of the photon momentum in the final state are constrained to discrete values. In this paper, different from that in [18], we will consider both the two factors in the electromagnetic transitions. Our result will show that the spontaneous emission rate depends on the position and the polarization of the atom. We will give a complete treatment which includes the influence on the transition amplitude in [20].

Consider the spontaneous emission in a cylindrical waveguide with radius R and length L_z ($L_z \gg R$). In the waveguide, some components of the photon momentum in the final state are constrained to discrete values, i.e., the number of the possible values of the photon momentum is less than that in free space. This means that the phase space volume of the final state is smaller than that in free space because of the existence of the boundary. More explicitly, in the cylindrical waveguide the component of the photon momentum which is perpendicular to the axis of this waveguide must satisfy

$$k_r^{TM} = \frac{j_{mn}}{R}, \qquad k_r^{TE} = \frac{j'_{mn}}{R}, \tag{1}$$

where j_{mn} is the n-th root of the m-th order Bessel function, j'_{mn} is the n-th root of the derivative of the m-th order Bessel function; the superscripts TM and TE denote TM and TE modes of the electromagnetic field in the waveguide, respectively. Therefore, to describe the state of a photon in the waveguide we need four quantum numbers: m, n, the z-component of the momentum k_z, and the polarization of the photon σ ($\sigma = TM$ or TE), and we will denote the state of a photon by $\mathbf{u}_{mnk_z\sigma}$.

For the spontaneous emission in the waveguide, an atom transfers from an initial state $|a\rangle$ into a final state $|b\rangle$ and produces a photon with frequency ω_p and mode $\mathbf{u}_{mnk_z\sigma}$. The S-matrix element to first order in perturbation theory is (in Heaviside's units and taking $\hbar = c = 1$) [21]

$$S_{ba} = -i2\pi\delta(E_b + \omega_p - E_a)\frac{1}{\sqrt{\pi R^2 L_z}} f_{ba}(\mathbf{r}), \tag{2}$$

where E_a and E_b are the energies of the two states of the atomic system, $f_{ba}(\mathbf{r})$ is the transition amplitude. Under the dipole approximation,

$$f_{ba}(\mathbf{r}) = -ie\sqrt{\frac{\omega_p}{2}}\mathbf{u}^*_{mnk_z\sigma}(\mathbf{r}) \cdot \mathbf{r}_{ba}, \tag{3}$$

where $\mathbf{r}_{ba}$ is the matrix element $\mathbf{r}_{ba} = \langle b|\mathbf{r}|a\rangle$. From the form of the transition amplitude (3) we can immediately find that the spontaneous emission rate must be dependent on the position of the atom, which is different from the case in free space.

The total spontaneous emission rate in this case can be obtained by summing over all final photon states:

$$
\begin{aligned}
W &= \frac{1}{\pi R^2}\sum_{m,n,\sigma}\int \delta(E_b+\omega_p-E_a)|f_{ba}(\mathbf{r})|^2 dk_z \\
&= \frac{\omega}{\pi R^2}\sum_{m,n,\sigma}\int \frac{1}{k_0^\sigma}[\delta(k_z-k_0^\sigma)+\delta(k_z+k_0^\sigma)]|f_{ba}(\mathbf{r})|^2 dk_z \\
&= \frac{e^2\omega^2}{\pi R^2}\sum_{m,n,\sigma}\frac{1}{k_0^\sigma}|\mathbf{u}^*_{mnk_z\sigma}(\mathbf{r})\cdot\mathbf{r}_{ba}|^2, \qquad (4)
\end{aligned}
$$

where k_0^σ is the z-component of the wave vector $\mathbf{k}$: $k_0^\sigma = \sqrt{\omega^2 - k_r^{\sigma 2}}$. We know that the total spontaneous emission rate in free space is $W_{free} = e^2\omega^3/(3\pi)|\mathbf{r}_{ba}|^2$ [21]; the ratio of the spontaneous emission rates in confined space and in free space, $\eta = W/W_{free}$, can be obtained directly

$$
\eta = \frac{3}{\omega R^2}\sum_{m,n,\sigma}\frac{1}{k_0^\sigma}\frac{|\mathbf{u}^*_{mnk_z\sigma}(\mathbf{r})\cdot\mathbf{r}_{ba}|^2}{|\mathbf{r}_{ba}|^2}. \qquad (5)
$$

This result implies that the ratio η is related with two scales: the radius of the waveguide R and the wavelength of the photon λ. Introducing a parameter $\xi = R/\lambda$, we can rewrite (5) as

$$
\eta = \frac{3}{2\pi\xi}\sum_{m,n,\sigma}\frac{1}{\sqrt{4\pi^2\xi^2 - j_{mn}^{\sigma 2}}}\frac{|\mathbf{u}^*_{mnk_z\sigma}(\mathbf{r})\cdot\mathbf{r}_{ba}|^2}{|\mathbf{r}_{ba}|^2}, \qquad (6)
$$

where $j_{mn}^{TM} = j_{mn}$ and $j_{mn}^{TE} = j'_{mn}$.

This result shows that the spontaneous emission rate of an atom in a waveguide is related to the position and the polarization of the atom. In free space, the mode of electromagnetic is homogeneity and isotropy, so the spontaneous emission rate of an atom is independent of the position and the direction of the atom. In a waveguide, however, the existence of the boundary spoils the symmetries of the mode, so the electromagnetic transition depends on the position and the polarization of the atom. In [18], the influence of the boundary to the mode of electromagnetic field is ignored; the mode is treated as the same as that in free space. In other words, the cavity mode $\mathbf{u}_{mnk_z\sigma}$ is approximately replaced by the plane wave, so the result must be independent of the position and the polarization.

On the other hand, from (6), the spontaneous emission rate can also be modified by changing the radius of the waveguide. Often, the spontaneous emission rate becomes smaller in a waveguide since the phase space volume of the final state becomes smaller; however, when ξ takes some special values, $\xi = j^\sigma_{mn}/2\pi$, the ratio η will diverge, i.e., the spontaneous emission rate will become extremely large, which means that a resonance will occur in the atom-cavity system. In addition, when the size of the waveguide is so small that the photon in the final state cannot exist, the spontaneous emission is forbidden completely. These effects imply a way to control the spontaneous emission rate by adjusting the size of the waveguide in which the atom placed.

3 The Influence of Non-Ideal Conducting Surfaces

In this section, we will consider the influence of a realistic, non-ideal conducting waveguide to the spontaneous emission of atoms. In a realistic waveguide, this influence will make the situation very different from the ideal case [22]. In fact, when an atom is very close to a non-ideal surface, one finds a massive increase in spontaneous emission due to non-radiative processes. This effect will cause decoherence in quantum information processing, and it needs to be avoided.

Compared with the ideal case, the influence of the non-ideal waveguide on the spontaneous emission rate of an atom mainly shows at the following two aspects. (1) When an atom is close to a non-ideal conducting surface, the spontaneous emission rate will increase rapidly. In an ideal waveguide, when the frequency of the photon emitted by an excited atom is lower than the lowest cutoff frequency of the waveguide, such spontaneous emission will be forbidden. On the contrary, in a non-ideal waveguide, when the atom is close to the wall, the spontaneous emission rate will increase as z^{-3} [23], where z denotes the distance between the atom and the wall. This means that in a realistic waveguide, the spontaneous emission which is forbidden in an ideal waveguide will not only become possible, but also may be greatly enhanced as long as the distance between the atom and the wall is small enough. (2) The curve of the spontaneous emission rate is softened. When placed in an ideal waveguide, the spontaneous emission rate of an atom can take the value of zero or infinite, but in a waveguide with non-ideal conducting walls, the spontaneous emission rate cannot be exactly zero or infinite, so the curve will be softened. However, this change will not greatly influence our result, since in a metal waveguide, the effects of suppression and resonance are still evident, then the situation will not greatly changed compared with the ideal case.

Spontaneous emission will lead to decoherence in quantum computation, so it should be avoided as much as possible. In the above mentioned two factors, the first one is obvious the more important one to cause decoherence, so we mainly need to overcome this problem.

When an atom is close to a non-ideal conducting surface, the interaction between the atom and the surface will become strong. Due to the extra phase shift of the reflected field, this interaction will enhance the spontaneous emission. For a very small distance between the atom and the surface, the spontaneous emission rate will remarkably increase. This influence makes the spontaneous emission rate change to [23]

$$\Delta W = W_{free}\left(\frac{\delta}{z}\right)^3. \tag{7}$$

Here

$$\delta = \frac{\lambda}{2\pi}\sqrt[3]{\frac{1}{4}\,\mathrm{Im}\left[\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+1}\right]}, \tag{8}$$

where λ is the wavelength of the emitted photon, $\varepsilon(\omega)$ is the dielectric constant of the waveguide wall at frequency ω. For the ideal case the conductance of a conductor is infinity, then $\delta = 0$ and $\Delta W = 0$. However, for a non-ideal conductor, its conductance is finite, so the parameter δ will not vanish, and when the distance $z \to 0$, the change of the spontaneous emission rate $\Delta W \to \infty$.

In the following, we will estimate the influence of the non-ideal effect on the spontaneous emission rate. From (7), in a non-ideal waveguide, the spontaneous emission rate will be remarkably influenced when the distance between the atom and the wall $z < \delta$; when $z > \delta$ the spontaneous emission rate can be approximately regarded as unchanged. Therefore the range that the influence of the non-ideal conducting surface dominates can be approximately described by δ. According to (8), this distance δ is determined by the dielectric constant of the conductor at a certain frequency. For the low-frequency case, e.g., in the microwave

Table 1 The values of the refractive indexes and δ for some metals at the wavelength $\lambda = 5893$ Å (Sodium D lines). The refractive indexes are taken from [25]

	Ag	Al	Au	Zn	Cu	Pb
n	0.20	1.44	0.47	1.93	0.62	2.01
κ	3.44	5.23	2.83	4.66	2.57	3.48
δ/λ	0.029	0.033	0.047	0.039	0.056	0.049

regime, metals are almost ideal conductors, so the influence of the walls can be neglected, i.e., $\delta \to 0$. For the high-frequency case, e.g., in the visible light regime, the refractive index and the dielectric constant both are complex. Denoting the refractive index as $n + i\kappa$, we can express the dielectric constant as [24]

$$\varepsilon = (n^2 - \kappa^2) + i2n\kappa. \tag{9}$$

Thus, by use of (8) and (9) we can calculate the range in which the spontaneous emission of excited atoms is remarkably influenced by the wall. In Table 1 we list the refractive indexes and the corresponding δ for some metal materials at the wavelength $\lambda = 5893$ Å (Sodium D lines).

From the results in Table 1, we can see that the distance δ for most metals hardly exceeds 5 percent of the wavelength at the visible light regime. It means that only when the atom is very close to the wall, about 0.05λ, its spontaneous emission rate will be remarkably changed. As a result, if the spontaneous emission is forbidden in an ideal waveguide, we can approximately regard that, as long as the distance between the atom and the wall is not too small, the spontaneous emission rate is unchanged when the atom is placed in a realistic waveguide with the same size. This result provides a method to reduce the spontaneous emission in a realistic waveguide: For preserving the distance between the atom and the walls, we can cover the inner surfaces of the waveguide by a thin film of some transparent insulating medium whose thickness is larger than δ. In such a waveguide, the spontaneous emission of atoms will still be suppressed, and so does the decoherence. Of course this method is also suitable for the lower frequency case, in which the influence of the non-ideal conducting walls is slighter and the insulating film can be thinner.

4 To Reduce Decoherence with the Help of Waveguides

In quantum information processing, one of the most severe obstacles is decoherence, which is caused by the interactions between the system and its environment. In a quantum computer the qubit is stored in a single particle, and crucially depends on the survival of quantum mechanical superposition states. Only in an absolutely isolated system, the quantum coherence of states can be maintained. However, any physical state cannot be made perfectly pure; it will always have some interactions with the environment. To avoid decoherence is therefore an essential and fundamental task in the realization of quantum computation.

Spontaneous emission is one of the most important sources of decoherence in quantum information processing, and it should be avoided. In the schemes of quantum computation with cavity QED, e.g., the scheme in [2], the atom is simplified to be a two-level one, and the influence of the spontaneous emission between the other levels is not taken into account. However, an actual atom must be a multi-level one; the spontaneous emission between two of these levels is used to transfer the information between atom and cavity, but the spontaneous emission between other levels will lead to decoherence. In this paper, we will take the scheme in [2] as an example to show our method to avoid the decoherence caused by the spontaneous emission: Use a waveguide with the size comparable to the wavelength of the photon emitted by the atom to replace the Fabry-Perot cavity or the confocal cavity, and the waveguide is made of metal and coated by a thin film of transparent insulating medium in the inner surfaces to avoid the decoherence caused by the interaction between the atom and the walls of the waveguide. We will show that, in principle, in our method any multi-level atom can be used and it will not cause extra decoherence. We can choose any atom with two appropriate levels to represent the qubit, these two levels are called work levels, even if there are other levels or the original lifetime of the work level is short.

The Ideal Case As discussed in last section, in an ideal waveguide the spontaneous emission rate of an excited atom is tightly related to the size of the waveguide: When the size of the waveguide takes such a value that the frequency of the photon in the final state is lower than the lowest cutoff frequency, the spontaneous emission is forbidden; when the waveguide resonates with the atom, the spontaneous emission rate will become extremely large. According to these properties, in information transmission, we can use a waveguide with a small enough size to avoid all spontaneous emissions, so the information will not be lost when the atom travels across the waveguide. In information transfer, we can use a waveguide resonating with the atom so that the spontaneous emission between the work levels (the levels which are used to represent the qubit) becomes very strong, and the spontaneous emission between any other energy levels will be forbidden. Therefore, the coupling between atom and cavity is made very large while the other spontaneous emission causing information loss is forbidden. Both of these two factors suppress the decoherence. Note that this proposal is not a concrete scheme of quantum computation; it is a general method which can be applied to many schemes and improve the efficiency of them.

More concretely, we take the scheme in [2] as an example. In this scheme a 2-bit universal quantum gate is outlined based on cavity QED: (1) A control bit a (represented by an atom) transfers its state to a cavity via the on-resonant interaction, i.e., the cavity resonates with the atom. (2) Another atom b is sent through the cavity and undergoes a conditional phase shift via the off-resonant interaction. (3) The state of the control bit is transferred back to an atom. In this scheme the interaction with the environment causes two kinds of decoherence: The interaction between the photon and the cavity leads to the cavity loss; the interaction between the excited atom and the vacuum fluctuation of the electromagnetic field leads to the spontaneous emission of the atom. To preserve the quantum coherence, a cavity with a high quality factor Q is used, and the cavity is adjusted exactly to resonate with the atomic transition. A high Q cavity makes the decay rate of cavity κ small; the resonance between the cavity and the atom makes the cavity-atom coupling g large, so the coupling between them is much faster than the loss of the information and therefore the decoherence can be suppressed. In this scheme, the atom is simplified to be a two-level one and the decoherence caused by spontaneous emission of the atom is not taken into account; however, a realistic atom must have many energy levels. For a multi-level atom, various kinds of spontaneous emission may take place, but only the one between the work levels is useful to transfer information; the others only cause the decoherence and should be avoided. For a multi-level atom, the spontaneous emission causes the decoherence at two stages in the scheme [2]: information transmission and information transfer. In information transmission, the information is storied in the state of the atom, so any spontaneous emission will cause the loss of information and any spontaneous emission should be eliminated. In information transfer, the information storied in the control atom a will be transferred to the cavity by spontaneous emission; this spontaneous emission is necessary and should be enhanced. However, any other spontaneous emission in which the atom transits to any other level causes decoherence and should be eliminated. Moreover, when the atom b passes through the cavity, it may spontaneously emit and this also should be eliminated.

To apply our proposal to the scheme suggested in [2] will obtain a more efficient quantum gate, in which the device consists of waveguides: In information transmission the atoms travel across waveguides, and the cavity is also replaced by a waveguide. In this improved scheme, the quantum information is represented by multi-level atoms. For control atom a, we choose the ground state and an excited state with suitable energy as the work levels, and the excited state need not be the first excited state. The spontaneous emission between the two work levels will be used to transfer the information, and the photon emitted by any other way of spontaneous emission has a lower frequency. For atom b, we also choose the ground state and an excited state as the work levels, but the energy difference between the two work levels must be smaller than that of atom a. In information transfer, we choose the waveguide having such a size that its lowest cutoff frequency exactly equals the frequency of the photon emitted by the spontaneous emission between the work levels of atom a. This resonance makes the coupling between atom and waveguide very strong, but all other spontaneous emissions cannot take place because the emitted photons have the frequencies lower than the lowest cutoff frequency of the waveguide. When atom b enters the waveguide, since the energy different of its work levels is smaller than that of atom a, the spontaneous emission between them is forbidden. Furthermore, any other possible spontaneous emission of atom b has even lower energy, and so is also forbidden. In information transmission, the atoms are placed in the waveguides whose sizes are smaller than that in information transfer. As a result, any spontaneous emission of any atom is forbidden, and the decoherence caused by spontaneous emission is in principle avoided. Therefore, compared with the scheme suggested in [2], replacing the Fabry-Perot cavity by a waveguide provides a more efficient method to avoid the decoherence caused by the spontaneous emission. Meanwhile, this proposal still has all of the advantages in the original scheme: (1) The waveguide can also resonate with the atom; it makes the coupling between atom and cavity extremely strong, so choosing a waveguide resonating with the atom can obtain a large coupling constant g. (2) The waveguide also can be made by highly reflective mirrors to reduce the cavity loss, so the cavity decay rate κ can also be small.

The Non-Ideal Case In the above discussion we use an assumption that the waveguide is an ideal one, in which the spontaneous emission of an excited atom can be forbidden. In a realistic system, however, when the atom is close to the wall, the situation will be very different: the interaction between the atom and the wall will lead to a rapid increase in spontaneous emission rate. The enhancement of the spontaneous emission will of course lead to decoherence in the quantum information processing, and this effect will make the above scheme invalid since it relies on the suppression of the spontaneous emission to decrease decoherence. To avoid such decoherence, we need to make some modifications to eliminate the influence of the non-ideal walls.

From Table 1 we can find that for a metal waveguide, only when the distance between the atom and the wall is less than 5 percent of the wavelength, the spontaneous emission rate remarkably increases. To avoid the decoherence caused by this effect, we simply require that the distance between the atom and the wall is larger than such a distance. In a realistic system, this can be realized by coating the inner walls of the metal waveguide by a thin film of transparent insulating medium with the thickness of 5 percent of the wavelength or more. In such a waveguide, the distance of the atom and the wall cannot be too small, so we can regard that the spontaneous emission rate is still greatly suppressed. In other words, the metal waveguide in the above scheme is replaced by a waveguide with a thin insulating coating in the inner walls to avoid the spontaneous emission caused by the interaction between the atom and the walls.

On the other hand, in a realistic waveguide, the curve of the spontaneous emission rate will be softened. Compared with the ideal case, when the frequency of the photon is lower than the lowest cutoff frequency of the waveguide with quality factor Q, the spontaneous emission rate of an atom in the waveguide will be suppressed by a factor of order Q [18]. However, this influence is not serious, when quantum computation is realized in a realistic high Q waveguide, the spontaneous emission rate is still much smaller than that in free space or in a Fabry-Perot cavity, i.e., the decoherence still can be reduced much more.

As mentioned above, the effect of the strong suppression of spontaneous emission of an excited atom in a waveguide can be used not only in quantum computation, but also in almost all aspects in quantum information processing, e.g., preserving quantum coherence of superposition states. A qubit is represented by a two state system whose state can be expressed as a superposition state; however, once the state is prepared, the non-zero probability of spontaneous emission of the excited state will spoil the coherence of the state gradually. To overcome the spontaneous emission and to preserve the coherence of the state, we can place the atom into an appropriate waveguide, in which the spontaneous emission is in principle forbidden. In other words, a feasible way to protect the qubit from the decoherence in quantum communication and in quantum computation is to place the atom in an waveguide with a sufficient small size.

5 Discussions

In summary, based on the discussion on the spontaneous emission of an atom in a waveguide, we propose a method in which a waveguide whose size is comparable to the wavelength of the photon emitted by the excited atom is used to replace the Fabry-Perot cavity to further reduce decoherence in quantum information processing. However, in a realistic non-ideal waveguide, the interaction between the atom and the walls will lead to a massive increase in spontaneous emission when their distance is small enough. To eliminate the spontaneous emission caused by the non-ideal conducting walls, we also suggest to use the waveguide whose inner walls are coated by a thin film of transparent insulating medium. We analyse the advantages of this method and show that the spontaneous emission can be greatly suppressed in an appropriate waveguide but in a Fabry-Perot cavity or a confocal cavity, the spontaneous emission rate cannot be greatly changed since none of them has a lowest cutoff frequency. Therefore, using waveguides in quantum computation can further reduce the decoherence, and this proposal is a general framework which can be used in many aspects in quantum computation. Besides the cavity QED scheme, there are some other schemes for realizing quantum computation and some demonstrative experiments have been made to show the possibility of quantum computation, with nuclear magnetic resonance technique [26–28], trapped ions [29–31], quantum dots [32, 33], spin chains [34], and so on. In these schemes, as long as spontaneous emission is a source of decoherence, our method is suitable for preserving the quantum coherence of the quantum information. We hope that this proposal may be realizable with current technology and testable in the future experiments.

In this paper, we suggest to use waveguides to avoid the useless spontaneous emission of atoms. In this method any multi-level atom can be used to represent the qubit because by use of a appropriate waveguide, in principle, we can choose any excited state of the atom and the ground state as the work levels, and any other spontaneous emission is forbidden so the other levels do not cause any influence. This property also extends the range for choosing the material to represent the quantum information, e.g., a highly excited state or a state whose original lifetime is very short can be used as the working level in our method.

However, the emitted photon by spontaneous emission of an atom usually has a short wavelength, but metal waveguides are usually suitable in the microwave regime. For realizing quantum computation, we need a waveguide with the size comparable to the wavelength of the emitted photon. It may be achieved from the following two ways. (1) Use a molecule to replace the atom to represent the qubit, so the emitted photon has a lower frequency and a metal waveguide will be suitable. (2) Find an appropriate waveguide which can be used in the visible light regime. For example, a light waveguide may be realized by use of the photonic crystal, although in this case some corrections should be considered [35].

Finally, it is worth remarking that the change of the phase space of the final state is caused by the existence of a boundary, so the suppression of spontaneous emission can appear in various cavities. In this paper, we only take the cylindrical waveguide as an example, but, in fact, a waveguide with another shape or a resonator can also play the same role as the cylindrical waveguide. In other words, we can use another cavity with a cutoff frequency in quantum information processing to suppress the spontaneous emission of excited atoms, and to reduce the decoherence caused by the interaction between the system and its environment.

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